



D&D Resources Ltd
Mathematics Exam Preparation Made Easy

Complex Numbers - 91577

Practice External Assessments 3

COMPLEX NUMBERS

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NCEA 3 Maths

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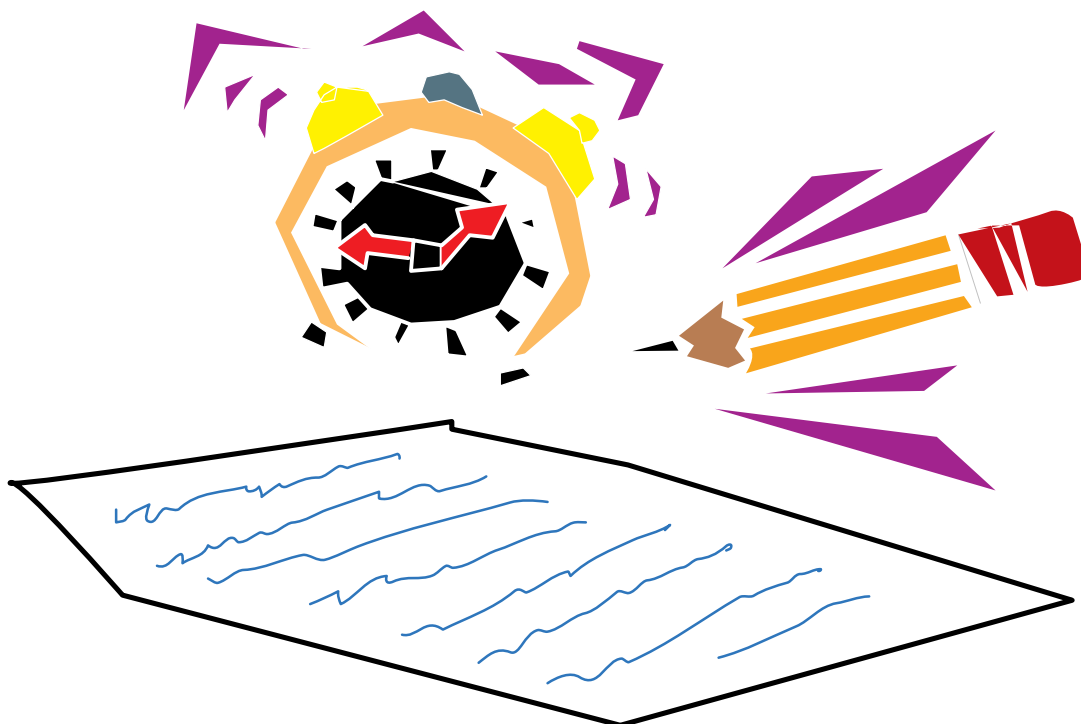
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Prior to attempting the practice assessments we suggest you download our Complex Numbers, 'Readiness Check', from the D & D Resources website. It comprises a series of questions together with answers which you can use as a 'warm up' before undertaking the assessments in this booklet.

The questions in the practice assessments are NOT in order of difficulty. Attempt all questions or you may not provide enough evidence to achieve the required standard.

Achievement Standard

91577

Apply the algebra of complex numbers in solving problems

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

- ◆ This achievement standard is derived from Level 8 of The New Zealand Curriculum and is related to the achievement objectives
 - ❖ manipulate complex numbers and present them graphically
 - ❖ form and use polynomial, and other non-linear equations in the Mathematics strand of the Mathematics and Statistics Learning Area.
- ◆ Apply the algebra of complex numbers in solving problems involves:
 - ❖ selecting and using methods
 - ❖ demonstrating knowledge of concepts and terms
 - ❖ communicating using appropriate representations.
- ◆ Relational thinking involves one or more of:
 - ❖ selecting and carrying out a logical sequence of steps
 - ❖ connecting different concepts or representations
 - ❖ demonstrating understanding of concepts
 - ❖ forming and using a model;
 and relating findings to a context, or communicating thinking using appropriate mathematical statements.
- ◆ Extended abstract thinking involves one or more of:
 - ❖ devising a strategy to investigate or solve a problem
 - ❖ identifying relevant concepts in context
 - ❖ developing a chain of logical reasoning, or proof
 - ❖ forming a generalisation;
 and using correct mathematical statements, or communicating mathematical insight.
- ◆ Problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or mathematical contexts.
- ◆ Methods include a selection from those related to:
 - ❖ quadratic and cubic equations with complex roots
 - ❖ Argand diagrams
 - ❖ polar and rectangular forms
 - ❖ manipulation of surds
 - ❖ manipulation of complex numbers
 - ❖ loci
 - ❖ De Moivre's theorem
 - ❖ equations of the form $z^n = r \operatorname{cis} \theta$, or $z^n = a + bi$ where a and b are real and n is a positive integer.

You should show ALL working and answer ALL parts of ALL questions.

(a) Simplify fully $\frac{a}{b+i} + \frac{a}{b-i}$.

(b) If $u = 2 \operatorname{cis} \frac{\pi}{k}$ and $v = 8 \operatorname{cis} \frac{2\pi}{k}$, write $\frac{v}{u}$ in polar form.

(c) The locus described by $|z-i| = |z-2|$ is a straight line. Find the y intercept of that line.

QUESTION TWO

- (a) Find the quadratic equation which has the roots $5 + \sqrt{2}i$ and $5 - \sqrt{2}i$.

- (b) If $u = 2k + i$ and $v = 2 + ki$, find the value of k if $uv = 9 + 20i$.

- (c) The quadratic $x^2 - 4$ is a factor of $3x^3 + ax^2 + bx - 8$. Find the values of a and b .

R such that $\frac{Px}{x+2} + \frac{Qx}{x-2} + \frac{R}{x^2-4} = \frac{8x-3}{x^2-4}$.

- (c) Find real numbers P, Q and R such that $\frac{Px}{x+2} + \frac{Qx}{x-2} + \frac{R}{x^2-4} = \frac{8x-3}{x^2-4}$.

Answers – 91577

PEA 2

Quest.	Evidence	Achievement	Merit	Excellence
		Apply the algebra of complex numbers in solving problems	Apply the algebra of complex numbers, using relational thinking, in solving problems	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems
ONE				
(a)	$\frac{a(b-i)+a(b+i)}{b^2+1} = \frac{2ab}{b^2+1}$	• correct solution.		
(b)	$4\text{cis}\frac{\pi}{k}$	• correct solution.		
(c)	$ x+(y-1)i = (x-2)+yi $ $\sqrt{x^2+(y-1)^2} = \sqrt{(x-2)^2+y^2}$ $x^2+y^2-2y+1 = x^2-4x+4+y^2$ $2y = 4x-3$ $y = 2x - \frac{3}{2}$ y intercept = $-\frac{3}{2}$	• correct equation line 2 or 3.	• correct equation and y intercept stated.	
(d)	$4(x-k) = 4x - 4\sqrt{x}\sqrt{k} + k$ $4x - 4k = 4x - 4\sqrt{x}\sqrt{k} + k$ $5k = 4\sqrt{x}\sqrt{k}$ $\sqrt{x} = \frac{5k}{4\sqrt{k}}$ $x = \frac{25k^2}{16k} = \frac{25k}{16}$	• correct expression for $\sqrt{x}$.	• correct solution.	
(e)	$z^2 = \frac{\sqrt{3}+i}{ai} \times \frac{i}{i} = \frac{-1+\sqrt{3}i}{-a}$ $= \frac{1}{a} - \frac{\sqrt{3}}{a}i$ $r^2\text{cis}2\theta = \frac{2}{a}\text{cis}\frac{-\pi}{3}$ $r^2 = \frac{2}{a}$ so $r = \sqrt{\frac{2}{a}}$ $2\theta = \frac{-\pi}{3}, \frac{5\pi}{3}$ so $\theta = \frac{-\pi}{6}, \frac{5\pi}{6}$ $z_1 = \sqrt{\frac{2}{a}}\text{cis}\frac{-\pi}{6}$ $z_2 = \sqrt{\frac{2}{a}}\text{cis}\frac{5\pi}{6}$	• expression for z^2 simplified.	• modulus or arguments correct or one solution only.	• both solutions correct.